22107305

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 5 May 2010 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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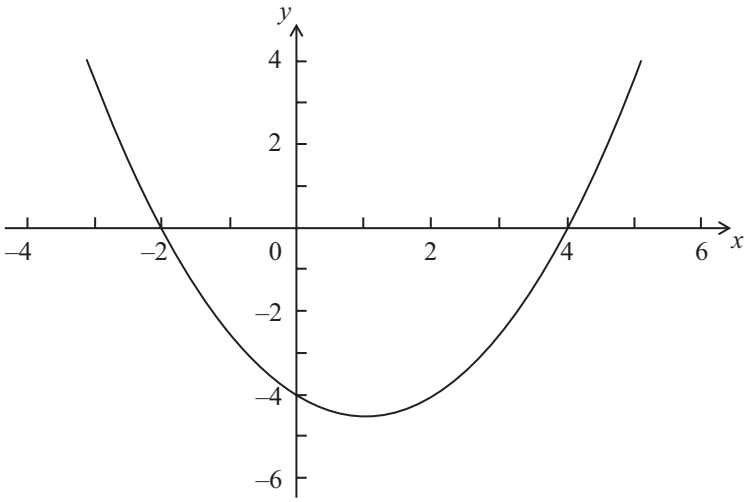
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

Let $f(x) = p(x-q)(x-r)$. Part of the graph of f is shown below.



The graph passes through the points $(-2, 0)$, $(0, -4)$ and $(4, 0)$.

(a) Write down the value of q and of r. *[2 marks]*

(b) Write down the **equation** of the axis of symmetry. *[1 mark]*

(c) Find the value of p. *[3 marks]*

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2. *[Maximum mark: 8]*

Let $\vec{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\vec{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

(a) Find $\vec{BC}$. *[2 marks]*

(b) Find a unit vector in the direction of $\vec{AB}$. *[3 marks]*

(c) Show that $\vec{AB}$ is perpendicular to $\vec{AC}$. *[3 marks]*

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3. *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$. *[3 marks]*

(b) Solve $\boldsymbol{A}^{-1}\boldsymbol{X} = \boldsymbol{B}$. *[2 marks]*

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4. *[Maximum mark: 7]*

Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

(a) Find $f\left(\frac{\pi}{2}\right)$. *[2 marks]*

(b) Find $(g \circ f)\left(\frac{\pi}{2}\right)$. *[2 marks]*

(c) Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$. *[3 marks]*

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5. *[Maximum mark: 6]*

Let $f(x) = kx^4$. The point $\mathrm{P}(1, k)$ lies on the curve of f. At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

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6. *[Maximum mark: 7]*

Solve $\log_2 x + \log_2 (x-2) = 3$, for $x > 2$.

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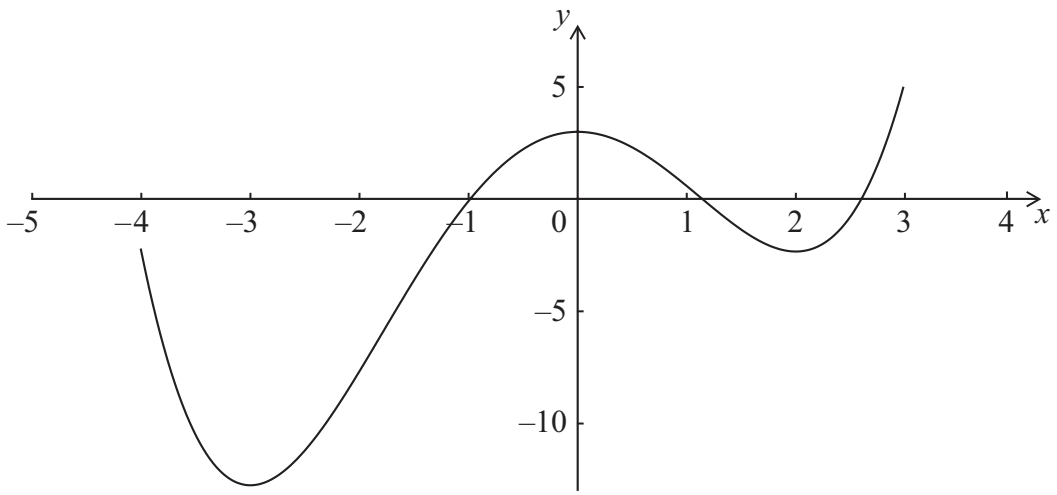
7. *[Maximum mark: 6]*

A function f is defined for $-4 \le x \le 3$. The graph of f is given below.



The graph has a local maximum when $x = 0$, and local minima when $x = -3,\ x = 2$.

(a) Write down the x-intercepts of the graph of the **derivative** function, f'. *[2 marks]*

(b) Write down all values of x for which $f'(x)$ is positive. *[2 marks]*

(c) At point D on the graph of f, the x-coordinate is -0.5. Explain why $f''(x) < 0$ at D. *[2 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider the function f with second derivative $f''(x) = 3x - 1$. The graph of f has a minimum point at $\mathrm{A}(2, 4)$ and a maximum point at $\mathrm{B}\left(-\frac{4}{3}, \frac{358}{27}\right)$.

(a) Use the second derivative to justify that B is a maximum. *[3 marks]*

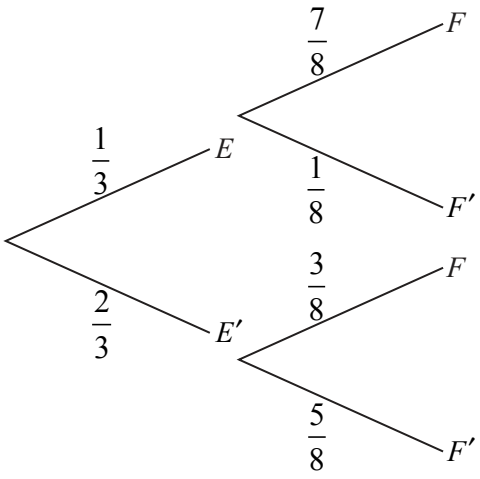
(b) Given that $f'(x) = \frac{3}{2}x^2 - x + p$, show that $p = -4$. *[4 marks]*

(c) Find $f(x)$. *[7 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 14]*

José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.
If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.
If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.
Let E be the event "he misses his bus" and F the event "he is late for school".
The information above is shown on the following tree diagram.



(a) Find

(i) $\mathrm{P}(E \cap F)$;

(ii) $\mathrm{P}(F)$. *[4 marks]*

(b) Find the probability that

(i) José misses his bus and is not late for school;

(ii) José missed his bus, given that he is late for school. *[5 marks]*

The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

(c) **Copy** and complete the probability distribution table. *[3 marks]*

***X* (cost in euros)**	0	3	6
P(*X*)	$\frac{1}{9}$		

(d) Find the expected cost for José for both days. *[2 marks]*

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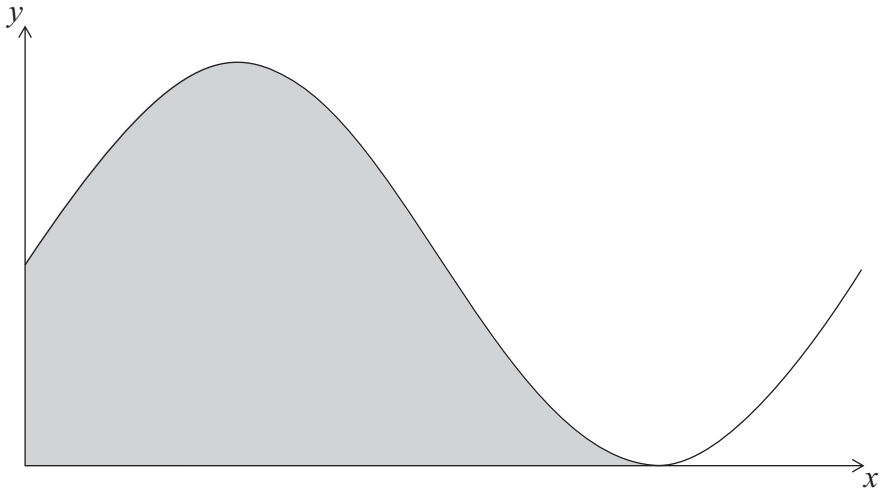
10. *[Maximum mark: 17]*

Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.



The shaded region is enclosed by the curve of f, the x-axis, and the y-axis.

(a) Solve for $0 \le x < 2\pi$

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$. *[5 marks]*

(b) Write down the exact value of the x-intercept of f, for $0 \le x < 2\pi$. *[1 mark]*

(c) The area of the shaded region is k. Find the value of k, giving your answer in terms of π. *[6 marks]*

Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g.

(d) Give a full geometric description of this transformation. *[2 marks]*

(e) Given that $\int_p^{p+\frac{3\pi}{2}} g(x)\,dx = k$ and $0 \le p < 2\pi$, write down the two values of p. *[3 marks]*